Cosmic Ray Muons

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In this experiment we explored the rate of incident cosmic ray muons on four paddle shaped scintillation detectors which were approximately 1 cm thick with an area of 900 cm. Detection rates were dependent on the polar angle the detectors were pointed. The efficiency of the detectors was also tested along with the coincidence rates between top detectors and bottom detectors. The muon lifetime was also measured by capturing rare events where a muon decays inside the detectors.

1. INTRODUCTION AND THEORY

Cosmic rays originating from outside our solar system, possibly from supernovae or other sources, constantly bombard our atmosphere. The constituent particles of cosmic rays, that is protons, alpha particles, and a small smattering of other light atomic nuclei, collide with the particles in Earth's atmosphere and produce a hadronic shower. These showers mostly consist of pions, $\pi^{0,\pm}$, that then decay into muons and neutrinos: $\pi^+ \to \mu^+ + \nu_\mu$ and $\pi^- \to \mu^- + \bar{\nu}_\mu$. The neutrinos that are produced are chargeless and nearly massless so they pass straight through earth without being detected very often. Muons, on the other hand, are about 200 times more massive than electrons ($m_\mu = 105.7$ MeV/c^2) and can be easily detected by many different kinds of techniques. Muons have an average lifetime of 2.197 μs[1] before they too decay into $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$ or $\mu^- \to e^- + \nu_e + \bar{\nu}_\mu$. Studying this decay process is how the muon lifetime portion of the experiment was determined.

Before diving into the muon lifetime, however, we were interested in measuring the muon flux, or the rate of muons that made it down to earth's surface and passed through our detectors. Muon flux determined by the following equation:

$$\frac{dN}{dt} = I_0 \cos^k(\theta) dA d\Omega \tag{1}$$

where $k \approx 2$ and $I_0 \approx 100$ m^{-2}sr^{-1}s^{-1}, θ is the polar angle that the detectors are pointed with respect to the vertical, dA is the differential area and $d\Omega$ is the differential solid angle; Figure 1 elucidates the quantities. We varied θ by tiling the metal frame that holds the detectors in 15° steps from 0° to 90°. Afterward, we varied the azimuthal angle ϕ as an extra check on whether or not the flux is changed.

Lastly, we measured the muon lifetime by capturing the signal of muons that decay inside our detectors. The probability density function for this occurrence is,

$$dP_e(t) = \Gamma e^{-\Gamma t} dt \tag{2}$$

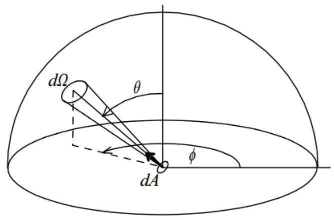

FIG. 1: This figure shows the differential quantities dA and $d\Omega$, and the polar angle θ and finally the azimuthal angle ϕ. We based the equations for flux on these quantities.

where t is the time for a single decay to occur and Γ is the inverse of the lifetime, $\frac{1}{\tau}$. We measured a distribution of these times which allowed us to discern the average lifetime of a muon.

2. APPARATUS AND EXPERIMENT

2.1. Counting Muons

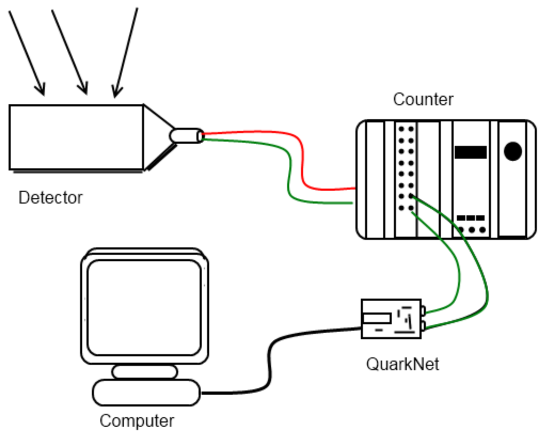

FIG. 2: An illustrated flow chart of the equipment. The chart shows the scintillation detector with its voltage line in red and its photomultiplier tube in green being fed to the logic gate that is the counter. Later in the experiment the counter signal was fed to the QuarkNet board and ultimately was read on the computer.

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The equipment setup for this experiment is depicted in Figure 2. The detector portion is made of polystyrene with a thin film of of florescent dye that emits a photon when a charged particle deposits energy on it. This set up employs four of these scintillating detectors in total with one pair (they are positioned on top of one another) at the top of a metal frame and the other pair at the bottom. The pairs are place approximately 115 cm apart which allows muons coming from 15.5° off the verticle to pass through all four of the detector paddles (see Figure 3). The frame allows the detectors to be tilted so that the normal of their surface can be pointed in different directions.

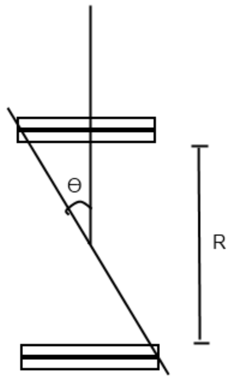

FIG. 3: Detector geometry where R = 114.72 cm and θ = 15.50°.

The detector paddles have two wires coming off of them; a red wire for power and a green wire which is a photomultiplier tube (PMT). We labeled the PMTs A, B, C, and D, where A is the top most detector and D is the bottom most detector. The PMTs are incredibly sensitive to light such that they can measure even a single photon. Lucky for us, because that is what this detection apparatus relies on for counting incident muons. We first investigated the count rate each individual channel had using the discriminator module. We found rates of between 50-80 counts/s whose fractional error was less than 2%. For counting statistics, the error in N counts is $\sqrt{N}$, so the fractional error is[2],

$$Fractional\ error = \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}}. \tag{3}$$

To achieve a fractional error of 2% or less this called for a an N of 2500 counts or more. The individual detectors needed a collection time of approximately 60 seconds to collect 2500 muons but on the next readings, where muon coincidence between detectors was measured, the collection time was adjusted to reach this threshold. The tabular summary of our measurements can be found in Appendix A.

2.2. Efficiencies

Occasionally a detector will miss a muon. For this reason it is necessary to check the efficiencies of each detector. This is done by using the LeCroy coincidence unit (part of "Counter" in Figure 2) which uses digital logic to determine if a muon passes through any combination of one, two, three, or four detectors. The efficiency of one detector was investigated by measuring the counts coincidence to all four detectors and dividing it by the counts coincident to the three other detectors that aren't the one being investigated. The logic is that if the three-fold coincidence counts some number more muons, it is fine to assume that the investigated detector in the four-fold missed that number of muons.

TABLE I: Detector efficiencies.

Detector	Efficiency	Uncertainty
A	0.925	.010
B	0.923	0.011
C	0.949	0.009
D	0.896	0.013

Multiplying the efficiencies of a combination of detectors tells the overall efficiency of the detectors in use. We found the the efficiency between for all the detectors was 0.7260, thus when finding the true count rate for all four detectors, the raw count rate is divided by this efficiency. This results in the average muon flux for combination AB decectors and combination CD detectors is 16.424 s^{-1}, this value will be used later in determining I_0.

3. ANALYSIS AND RESULTS

3.1. Angular Dependence

We measured the four-fold muon flux with respect to the polar angle θ and again with respect to the azimuthal angle ϕ. We let $\theta = 0$ be straight up through the roof of the building and $\phi = 0$ be East (out the window). When changing the polar angle the rate dips such that we had to increase the data collection time from 1000 seconds (16.67 minutes) at 0° to 3000 seconds (50 minutes) at 90°. Figure 4 shows the results of changing the polar angle from 0° to 90° while holding the azimuthal pointed East. The plot shows how the highest rate of muon interactions comes from directly above at $\theta = 0$.

For the fit we used the given equation,

$$R_B + R_0 \cos^k \theta \tag{4}$$

and found that $R_B = 0.030$, $R_0 = 0.754$, and $k = 1.798$. We suspected that the point at 0° was somewhat high

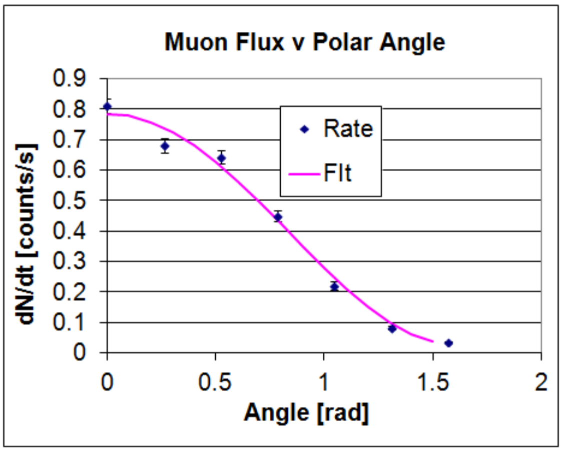

FIG. 4: The rate of the four-fold efficiency-corrected incidence as a function of polar angle. The χ^2 value of the fit is 24.6.

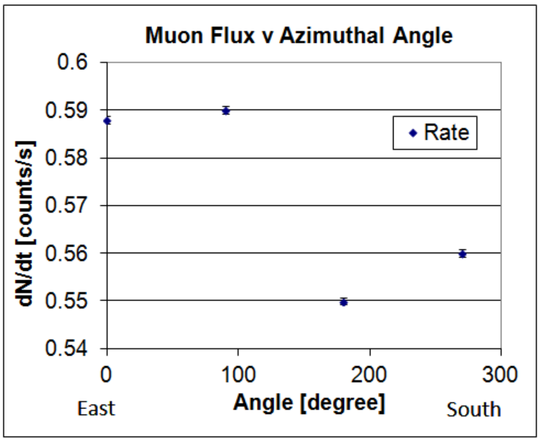

FIG. 6: The rate of the four-fold efficiency-corrected incidence as a function of azimuthal angle. We set the polar angle to 30° so that turning the frame would have an effect.

so we did the plot and fit again with it excluded. Ideally with 7 data points the χ^2 would be about 7, ours however was about 24. With the first point excluded, therefore 6 points, we achieved a χ^2 of around 14 which was somewhat better. In the second case, $R_B = 0.035$, $R_0 = 0.746$, and $k = 1.948$.

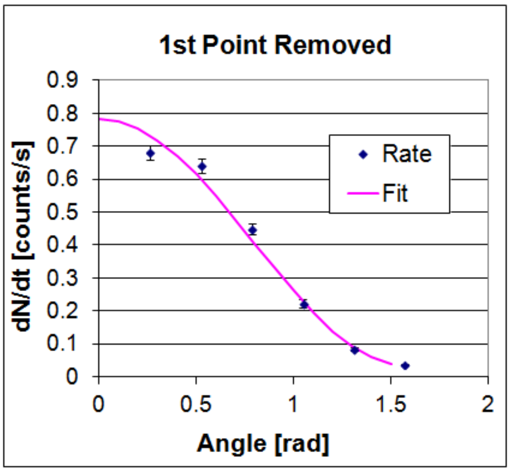

FIG. 5: The first plot of Muon Flux vs Angle for this experiment had a suspiciously high first entry. This plot shows the first point at polar angle 0° (the angle that goes points the detectors up through the building) and the new fit that results. The χ^2 value for this fit is 14.4.

Next, we took measurements of the muon flux while varying the azimuthal angle. Figure 6 shows that the count rate stays almost constant with just a small deviation. We expected that the North and South flux would be the same, the East (out the window) direction would be the highest and the West direction would be the lowest (point into the heart of the building) due to the "amount of building" that is above those directions. Another source of potential azimuthal changing in flux is the fact that μ^- and μ^+ are not produced at the same rate in the upper atmosphere[3], μ^- make up 55% of the muons produced and while μ^+ makes up only 45%. Earth's magnetic field may bend the tragectory of the muons in such a way that changing the azimuthal angle does present some anisotropy. However, change in flux with respect to the azimuthal is not large enough to interfere with this experiment, therefore we will say muon flux is approximately isotropic with respect to the azimuthal angle.

3.2. Muon Lifetime

Next, we hoped to make an accurate measurement of the muon lifetime from a distribution of muon decays that occur within the detectors. What we are measuring in the detector is the decay channel

$$\mu^- \rightarrow e^- + \bar{\nu_e} + \nu_e \tag{5}$$

because a muon will deposit energy and the dye film will scintillate, but then decay happens and the electron also deposits energy and causes a scintillation in a 20 μs window. Our experiment utilized a 20 ns internal clock that allowed us to have 1024 data bins spread over the 20 μs interval. Figure 7 shows the frequency of events that met the requirements of triggering and stopping the clock, namely, two pulses in less than 20 μs. While a muon decay will trigger a count, there are other processes that also trigger a clock start and stop.

Chief among the sources of background are the special cases where a muon hits the target and a second muon also hits the target within the specified interval. A second potential source of background is other particles that result from cosmic rays, however, they have a much smaller atmospheric abundance. A breakdown of the signal to background events is shown in Appendix A section 4.

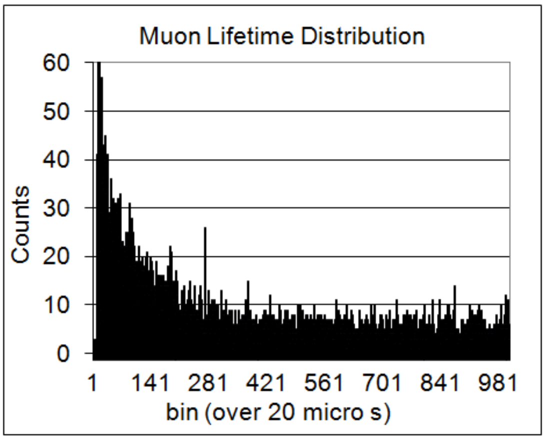

FIG. 7: Raw data for muon decay frequency where the x axis spans 20μs divided into 1024 bins. The large spikes (about 20k counts) that occur around 20ns have been excluded.

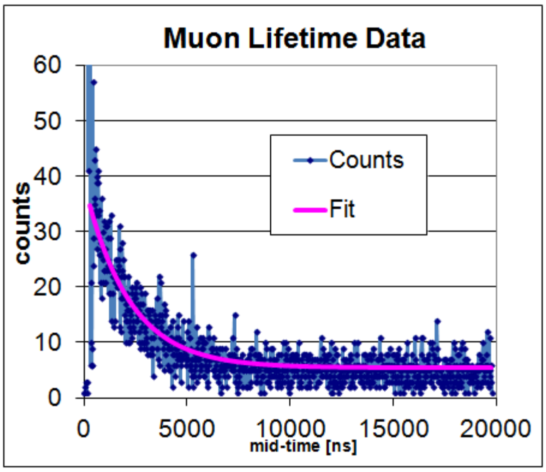

FIG. 8: The counts as a function of time where the x axis spans 20μs divided into 1024 bins. The pink shows the fit.

$$dP = \Gamma dt. \tag{6}$$

These events are Poisson distributed that have a probability of decaying in the next time dt that goes like,

Using this information, we fit our data to the equation,

$$\mu_i = A + Be^{-t_i/\tau} \tag{7}$$

where μ_i is the mean number of counts for each bin i, and τ is the muon lifetime. We used a χ^2 minimization technique (χ^2 = 1078) and a log likelyhood technique ($\chi^2_{Poisson}$ = -1755) which yielded the same results. For our fit, the background level term $A = 5.371$, the scaling term $B = 33.672$, and the muon lifetime

$$\tau = 2.169 \pm 0.050. \tag{8}$$

which is off by 1.55% from the NIST value.

4. CONCLUSIONS

We set up four scintillating detectors and found their muon detection efficiency. We used that efficiency to determine the true muon flux through the detectors. This rate was between 50 s^{-1} and 80 s^{-1} with the detectors in a vertical position. Then measurements for the muon flux as a function of the polar angle were taken, and it was determined that with $k = 1.798$, A = 0.09 m^2, and that the average rate of detection for AB and CD was 16.424 s^{-1}, and Equation 13 in the manual[4] gives,

$$I_0 = \frac{dN}{dt}\frac{k+2}{2\pi A} \tag{9}$$

then,

$$I_0 = 107.461 s^{-1} m^{-2} sr^{-1}. \tag{10}$$

Muon flux has a strong dependence on the polar angle θ for which we found $dN/dt \propto \cos^k\theta$. Conversely, muon flux has almost no dependence on the azimuthal angle ϕ, which is only subject to inhomogeneities in muon path.

Finally, by taking a distribution of muons that decayed inside our detectors we were able to extrapolate what the average muon lifetime is. As discussed above we found that it is $\tau = 2.169 \pm 0.050$. These measurements in conjunction with the true count rates of our detectors allowed us to have double confirmation of what events were background detections.

[1] NIST, *National institute of standards and technology. online database* (http://physics.nist.gov., 2013).
[2] R. Deserio, *Muon Lifetime Measurement CRM Addendum* (UF Physics Dept, 2012).
[3] Galbiati and Beacom, Physical Review C (2005).
[4] R. Deserio, *Cosmic Ray Muons* (UF Physics Dept., 2012).

1. CQ 1

If $\sigma_{\epsilon_i} = \eta$ if the uncertainty for something being counted, where

$$\eta^2 = \Sigma_{i=1}^{N} \sigma_i^2 \left(\frac{\partial \epsilon_i}{\partial y_i}\right)^2 \tag{11}$$

and $\sigma_i \approx \sqrt{N_i}$. For

$$\epsilon_1 = \frac{N_{+1}}{N_{+1}+N_{-1}} \tag{12}$$

Then we have

$$\frac{\partial \epsilon_1}{\partial N_{+1}} = \frac{N_{-1}}{(N_{+1}+N_{-1})^2} \tag{13}$$

$$\frac{\partial \epsilon_1}{\partial N_{-1}} = \frac{-N_{+1}}{(N_{+1}+N_{-1})^2} \tag{14}$$

And this means

$$\sigma^2_{\epsilon_1} = \frac{N^2_{-1}N_{+1}+N^2_{+1}N_{-1}}{(N_{+1}+N_{-1})^4} \tag{15}$$

$$= \frac{N_{+1}}{N_{+1}+N_{-1}}\frac{1}{N_{+1}+N_{-1}}\frac{N_{+1}+N_{-1}-N_{+1}}{N_{+1}+N_{-1}} \tag{16}$$

$$= \frac{\epsilon_1(1-\epsilon_1)}{N_{2+3+4}} \tag{17}$$

2. CQ 2

Differences in flux with respect to the azimuthal angle could most likely be due to the different conditions above the detector. For example, in our experiment, when the detectors are pointed East, they are pointing out the window and the incoming muons have no concrete to penetrate. The other cardinal directions all have, roughly, the two roofs for the muons to come through. So we must ask the question, how much change does the concrete of the building introduce? The density of the atmosphere is $\rho = P/gcos\theta$ were g is the acceleration due to gravity at Earth's surface and P is atmospheric pressure. Straight up, the areal density of the atmosphere is $\rho = 10,000kg/m^2$ and at a 45° it is $\rho = \sqrt{2}\cdot 10,000 = 14,000kg/m^2$. If concrete has a mass density of 2400 kg/m^2 then the atmosphere straight above equates to about 4 meters of concrete. If the roof is also 40 cm and the muons must pass through two, then the muons must pass through an equivalent of 4.8 meters of concrete where just the roofs make up 16% of that. This is a non-trivial amount.

3. CQ 3

Our data supports time dilation of the theory of Special Relativity because we predicted in Exercise 3 that practically all of the muons would decay before reaching Earth's surface. Figure 4 shows that this is clearly not the case. For a muon of about 4 GeV, the realtistic factor $\gamma \approx 40$, and so the muon would travel $\gamma\cdot$ 600 m = 26 km on average before decaying.

4. CQ 4

We obtained 2.169 ± .050 μs as our value for the muon lifetime. NIST lists the accepted value of the muon lifetime as 2.197 ± 0.00004 μs, which means our measurement is off by 1.55%. By ridding our measurement of the background counts, we determined that the total number of muons that stopped in our detectors was approximately 3200. The total muon flux was 8600, therefore 37.2% of the events that trigger a double pulse are actually muons decaying in the detectors. The two-fold muon flux for either pair of detectors was 16 /s and the data collection spanned over 92 hours, thus we had 5.3 million muons in total. The fraction that decays in the detectors is .06%.
In order to compare the background rate with the random coincidence rate, we use this equation (that Dr. Furic helped us derive in class),

$$N_{background} = T\Delta t\lambda^2 \tag{18}$$

where T = 331200 s is the data collection time, Δ t = 20 μs is the time interval for a valid double pulse, and λ = 32 /s is the muon flux for detectors AB (16 /s) plus the rate for CD (16 /s). Plugging in these values we get,

$$N_{background} = 5900 \tag{19}$$

which is rather close to our measured value of 5300 back ground events.

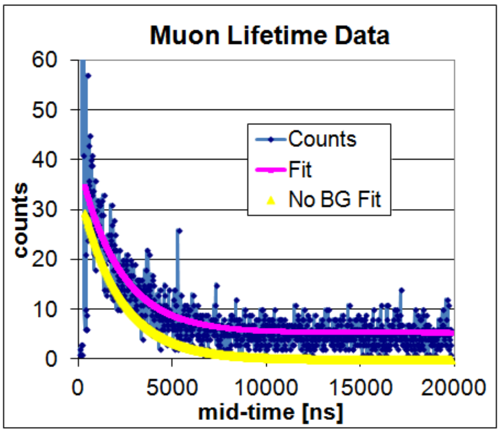

FIG. 9: Muon lifetime distribution where the yellow line shows the counts if no background events were included.

Ch	Time [s]	Counts	σ_Counts	Eff	σE	True Counts	True Rate [counts/s]	Prop. Of Error
A	60	4009	63.317	0.925	0.010	4334.054	72.234	84.1661
B	60	4020	63.403	0.923	0.011	4355.363	72.589	85.5406
C	60	3686	60.712	0.949	0.009	3884.089	64.735	72.9467
D	60	2629	51.274	0.896	0.013	2934.152	48.903	70.4061
AB	240	3392	58.241	0.854		3972.944	16.554	
CD	240	3325	57.663	0.850		3910.366	16.293	
AC	900	610	24.698	0.878		694.899	0.772	
BD	900	601	24.515	0.827		726.716	0.807	
AD	900	541	23.259	0.829		652.751	0.725	
BC	900	647	25.436	0.876		738.646	0.821	

FIG. 10: Coincidences and efficiencies.