Quartz Crystal Tuning Fork in Liquid Helium

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In this experiment we explore the behavior of a quartz crystal tuning fork in gaseous helium and liquid helium. The tuning fork acts as a resonator with a high quality factor, meaning it has a tall, thin peak at its resonant frequency. The purpose of this experiment was to observe how the resonant frequency and damping of the tuning fork change in different mediums. A cryostat was used to provide a low temperature environment in order to immerse the fork in gaseous, normal liquid helium, and super fluid helium.

1. INTRODUCTION AND THEORY

Our objective was to observe the behavior of a small tuning fork in different mediums. The tuning fork used in this experiment is only millimeters long and its tines are only a fraction of a millimeter in width.

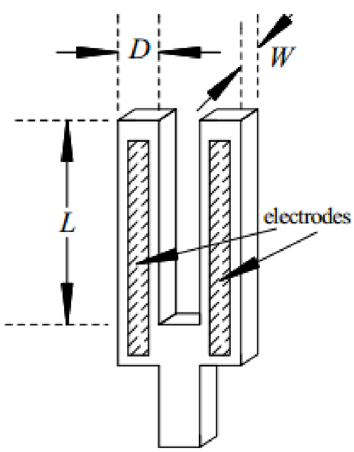

FIG. 1: Tuning fork schematic.

As shown in Figure 1, the fork is two rods that are connected at the bridge, which in itself is a complex system to model. The simplest mode of oscillation is when the tines move such that the base of the tine stays stationary and the tip sweeps out an arc. The movement of the tines can be modeled using equations of a driven harmonic oscillator whose equation is,

$$\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \omega_0^2 x = \frac{F}{m}cos(\omega t + \delta) \tag{1}$$

where $Fcos(\omega t + \delta)$ is the driving force, γ is the damping constant and ω_0 is the resonant frequency and is defined as,

$$\omega_0 = \sqrt{\frac{k}{m}}. \tag{2}$$

Quartz is piezoelectric, meaning it deforms if a voltage is applied to it. The fork is a monolithic quartz crystal and the tines are activated by putting a voltage through electrodes on them. The voltage cannot be too high or the fork will break because the deformation becomes too great and the amplitude of the arc that the tine sweeps out ends up crossing with the other tine, shattering the crystal. The amplitude of the oscillation is as follows,

$$A = \frac{F/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + \omega^2\gamma^2}}. \tag{3}$$

Putting the tuning fork in different mediums changes the resonant frequency and the damping constant. For example, a tuning fork in vacuum has a smaller damping constant than a tuning fork in air because air has more viscosity. This is the primary motivation for this experiment and in the following sections we will see the effects different mediums have on the tuning fork's properties.

2. APPARATUS AND EXPERIMENT

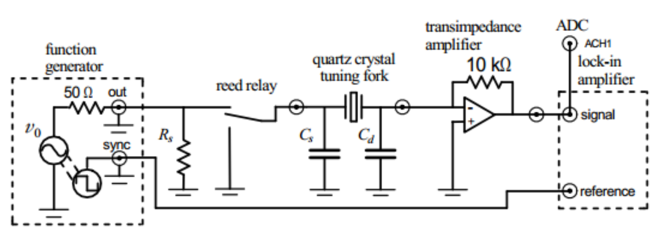

FIG. 2: This is the diagram for the circuit we used. The function generator and the lock in amplifier are delineated while the control box (trans-impedance amplifier) is between them.

The apparatus we used allows the tuning fork to be set for use in air, vacuum, and helium, as well as a resistor and a capacitor. Figure 2 shows the set up where the

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'quartz crystal tuning fork' label could be swapped out for the resistor or the capacitor by changing the dial on the trans-impedance amplifier (which is represented by the non-boxed area of Figure 2). Our initial measurements and observations were done on the resistor, this was mainly where we explored the set up and made certain everything was working properly.[1] There are three pressure gauges that tell us what the pressure is at different parts of the set up; the most important one was the Matheson gauge, which we used extensively.

While the tuning fork is a large component of the experiment, the bigger picture here is understanding cryostat and the behavior of helium. The suck stick apparatus is a double jacketed metal tubing that allows helium to pass slowly through a capillary when it is inserted into a dewar. The double jacket provides an insulating layer for the inner jacket so that a temperature of 1.6 K can be reached. Once inside the dewar which has liquid helium held at 4.2 K, the internal volume of the suck stick is pumped on.[2] The most energetic particles are the ones that have liberated themselves from the liquid and by pumping those out, it rids the volume of some of its heat. This process is what gets the temperature through the lambda point at 2.18 K and the helium becomes a super fluid.

There were three distinct stages of cooling down: between room temperature and 4.2 K, between 4.2 K and 2.2 K, between 2.2 K and 1.6 K. Because the capillary is slowly letting helium in, at 1.6 K it took twenty minutes for the tuning fork to become submerged. However, as instructed, as we waited for the cooling to happen we took measurements of ω_0 and γ as a function of pressure, as shown in Figure 3 and 4.

[1] We lost all of day 3 because the trans-impedance amplifier needed to be fixed.

[2] The schematic for the gas handling is located in Appendix A

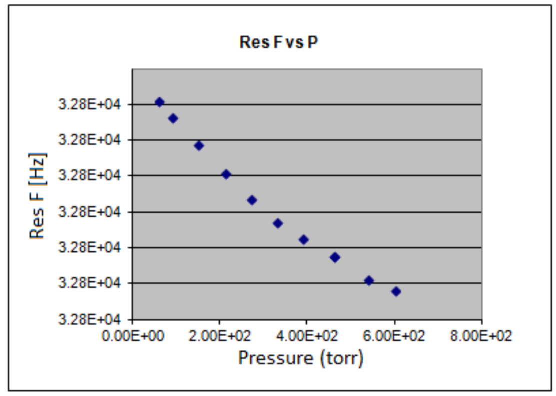

FIG. 3: Tuning fork resonant frequency as a function of pressure. The pressure was altered by opening value 6 and 7 and watching the Matheson gauge change. We had to be somewhat careful because the Matheson gauge is not an absolute gauge. It is relative to the pressure inside the of it that is pushing on a 'c' shaped mechanism that is supposed to be at atmosphere for reference. We noticed if the pressure changed too quickly the Matheson gauge would fluctuate before settling on a pressure.

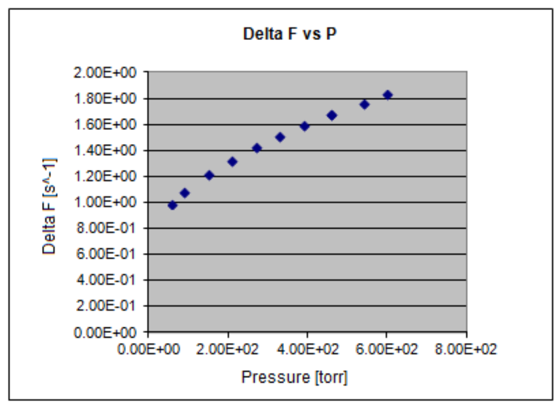

FIG. 4: Damping constant as a function of pressure.

These data make sense because they show that less pressure results in a higher resonant frequency and a smaller damping constant. If there is less pressure, then there is less gas in the cavity and therefore less drag from its viscosity. We will see what happens when the tuning fork is submerged in a viscosity free liquid, namely, super fluid helium. The following section focuses on the experiment when it neared the lambda point.

3. ANALYSIS AND RESULTS

Our primary interests were related to the resonant frequency as a function of temperature. The effects of a viscous medium are seen in the changes in mass and drag. In a viscous medium, the mass is "enhanced" and becomes,

$$m^* = \beta\rho V + B\rho S\lambda \tag{4}$$

where β and B are geometrically dependent constants and $S = 2(D + W)L$ is the surface area. The drag coefficient becomes,

$$b^* = \sqrt{\frac{\rho\eta\omega}{2}}CS \tag{5}$$

where C is another coefficient and η is the viscosity of the medium.

By some unclear way, the manual gets rid of the λ and points out ω can be swapped out for ω_0 because the frequency dependence is minimal [1]. Then these quantities turn into,

$$m^* = \beta\rho V + BS\sqrt{\frac{2\eta\rho_n}{\omega_0}} \tag{6}$$

and

$$b^* = \sqrt{\frac{\rho\eta\omega_0}{2}}CS \tag{7}$$

.

With an interest in keeping this to four pages, we fast forward to these equations the manual recommended plotting. Equation 8 shows the function $\mathcal{F}$ which is m^*/m.

$$\mathcal{F} = \frac{\beta\rho V}{m} + \frac{BS}{m}\sqrt{\frac{2\eta\rho_n}{\omega_0}} \tag{8}$$

Meanwhile, m^*/m is also equivalent to,

$$\mathcal{F} = (\frac{\omega_{00}}{\omega_0})^2 - 1 \tag{9}$$

where ω_{00} is the resonant frequency in vacuum. We measured ω_{00} to be 32.685 kHz. Figure 5 shows what equation 8 looks like with all the constants gathered from the class resources [?]. Figure 6 shows equation 9 plotted with the experimentally gathered values of ω_{00} and ω_0.

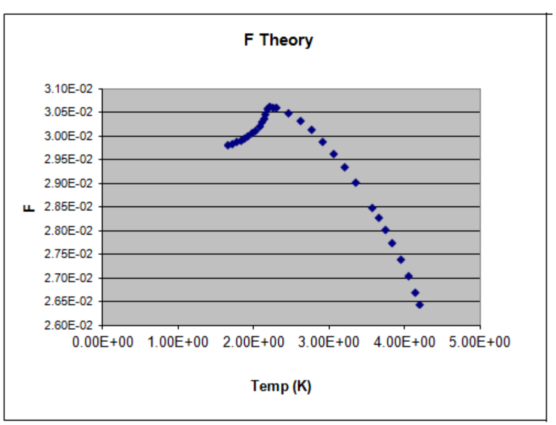

FIG. 5: Expected F from theory as a function of temperature.

The fit for the experimental data was very good. We found that $\beta = .130$ and $B = .171$. Similarly, however,

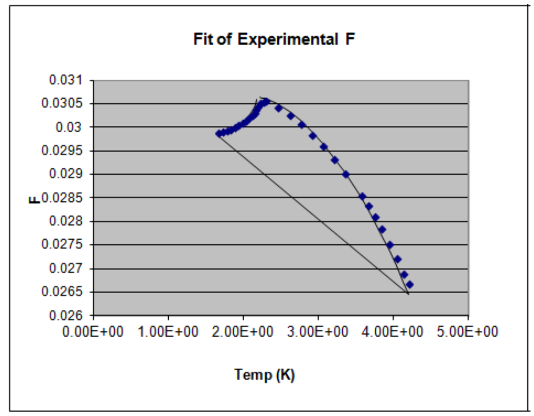

FIG. 6: Fit of experimental data for F as a function of temperature.

concerning the damping constant of the system, b^*/m is equivalent to,

$$\mathcal{G} = \sqrt{\frac{\rho_n\eta\omega_0}{2}}\frac{CS}{m} \tag{10}$$

and

$$\mathcal{G} = \gamma(\frac{\omega_{00}}{\omega_0})^2 - \gamma_0 \tag{11}$$

where γ_0 is the damping constant in vacuum that we measured as .318 s^{-1}. The plots of these functions are shown below.

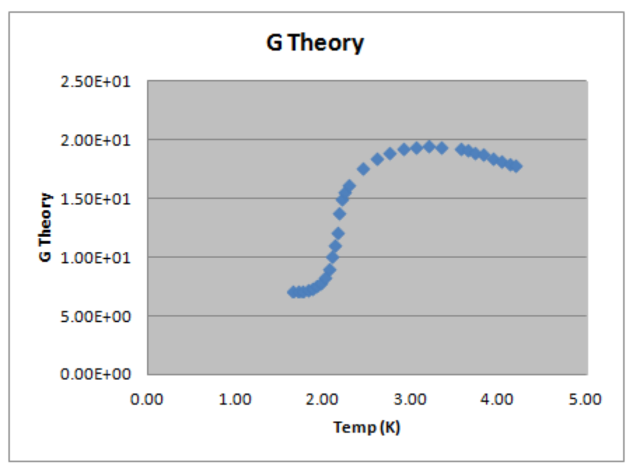

FIG. 7: Expected G from the theory as a function of temperature.

The fit of the experimental data for $\mathcal{G}$ was really bad. There is a dip in the data at 3.4 K that is not supposed to be there, however, before and after that the data seems ok. The lowest temperature data does not quite match, however that is most likely explained by where equation 10 breaks down. We obtained that $C = .40$. $\mathcal{G}$ shows us clearly that the damping constant takes a huge nosedive

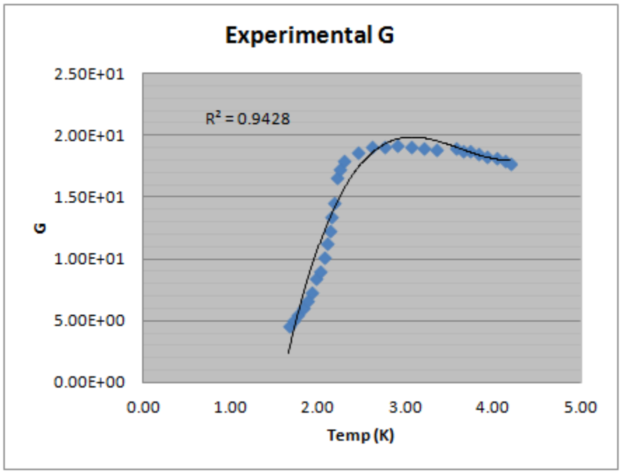

FIG. 8: Fit of experimental data for G as a function of temperature.

below 2.2 K which is coincident with the transition the helium makes into a super fluid state.

4. CONCLUSIONS

The data exhibited above shows us that the resonant frequency and damping constant are affected by the type of medium that the resonator is in. By analyzing functions like $\mathcal{F}$ and $\mathcal{G}$ we are able to see the effects more clearly, and, most importantly, make sense of the changes theoretically based on the system's physical quantities like density of the liquid, mass of the tuning fork, viscosity, and the temperature.

The manual asked us to find out why pressure has no effect on the viscosity of gaseous helium. Viscosity is independent of pressure because viscosity is a function of density, the mean free path and other variables. But mean free path is actually inversely proportional to density. So even if you increase the pressure, the change falls out in the end. [2]

This experiment could further investigate the mass accretion over time as the tuning fork cools down. It was suspected that there was gas condensing on the tines and changing both the damping constant and the resonant frequency in a way that could not be accounted for at the time.

[1] R. J. Donnelly and C. F. Beranghi, *The observed properties of liquid helium at the saturated vapor pressure* (Phys. and Chem. Ref. Data, 27, 1217- 1274, 1998).

[2] *Viscosity info from wikipedia*, URL http://en.wikipedia.org/wiki/Viscosity.

[3] R. Deserio, *Quartz Crystal Tuning Fork in Super uid Helium* (UF Physics Dept., 2012).

Appendix A: Comprehension Questions

1. C.Q. (page 22)

Compare the results of the frequency scans and ring downs and discuss their pros and cons for determining 0 and . Describe how to determine the motional resistance R and do so for both tuning forks in the interface box.

While both frequency scans and ring downs were possible ways to determine ω_0 and γ, each had their problems and benefits. The frequency scan would have been best to use if there had been secondary or tertiary peaks to observe. But because there were not, an accurate scan ended up having an excruciatingly long run time (about 15 minutes) to do something that could be achieved much more quickly. Enter ring downs. Ring downs were a quick way to get the same information about ω_0 and γ.

The motional resistance R is defined as,

$$R = \frac{2b}{\kappa^2} \tag{A1}$$

so we need b and κ. This is a complicated question so lets see what cards we have to play.

• Exercise 3b already found that a tuning fork in vacuum has $b = 6.8 \times 10^{-8}$ Hz ·kg.

• Exercise 5b found that a fork in vacuum has a $\kappa = 3.65 \times 10^{-6}$ C/m.

So for a tuning fork in vacuum the motional resistance is $R \approx 11$ kΩ.

If κ is indeed dependent "on the tuning fork geometry, the cut of the tuning fork relative to the crystal axes of the quartz, and the electrode shape and placement" [3] then I see no problem with using the same value for the tuning fork in air. In this case, some of our preliminary measurements will come in handy, see Figure 9. A crude estimate of the FWHM is about 6 Hz. In which case for γ much less than ω_0 we can use, from Exercise 3,

$$FWHM = \frac{\sqrt{3}\gamma}{2\pi} \tag{A2}$$

$$\implies \gamma = 21.7 \tag{A3}$$

$$\implies b = 1.6 \times 10^{-6} \tag{A4}$$

and these values suggest the motional resistance is $R \approx 244$ kΩ for a tuning fork in air.

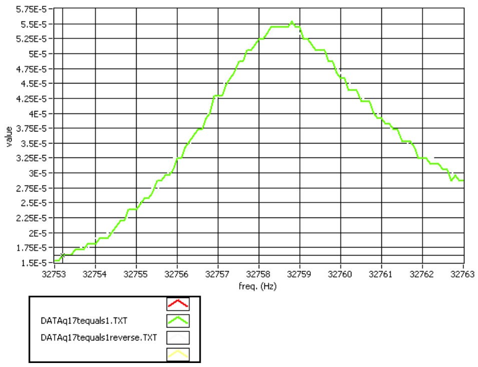

FIG. 9: Data acquired during the early measurements. This image shows the tuning fork in air that is located in the interface box.

2. Page 30

1. Summarize the data and explain what you learned in the initial observations.

This is a silly question because it is answered in the Apparatus and Experiment section.

2. Make plots of f_0 and δf vs. T and of F and G vs. T and t the latter two to the theory. Are the deviations reasonable? Do you see any evidence of systematic differences between the theory and the data?

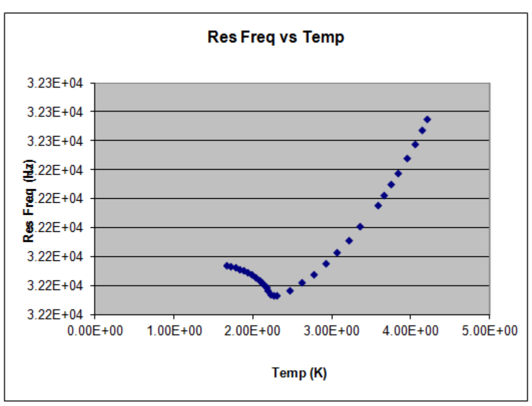

FIG. 10: Data acquired during the early measurements. This image shows the resonant frequency of the tuning fork as the helium made its transition between super fluid and normal.

There seem to exist some systematic differences between the theory and the data when it comes to the damping

3. How does a liquid at the bottom of a long column of liquid helium cool down when the pressure at the top

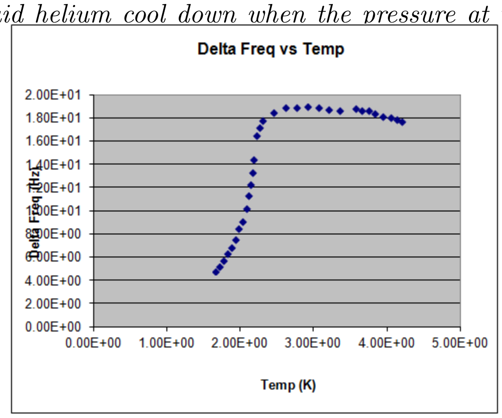

FIG. 11: Data acquired during the early measurements. This image shows the damping constant as the helium made its transition.

is reduced? How does the liquid at the bottom warm up when the pressure is increased?

The liquid at the bottom of a long column of liquid helium cools down when the pressure is reduced because the partial pressure of the liquid is changed. If the pressure at the top of the column is reduced then helium atoms are more likely to liberate themselves from the liquid. This is an example of evaporative cooling, the helium atoms that leave the liquid are carrying away heat and that is a reduction of entropy. Thus through thermal conduction the liquid at the bottom of the column will cool. This is the mechanism pumps use to cool things down, they pump away the most energetic particles.
Similarly, if the pressure that the top of the column is increased then helium atoms that are in their gaseous state are more likely to rejoin the liquid and with them carrying energy into the liquid. By the same logic, this process warms the liquid by conduction.

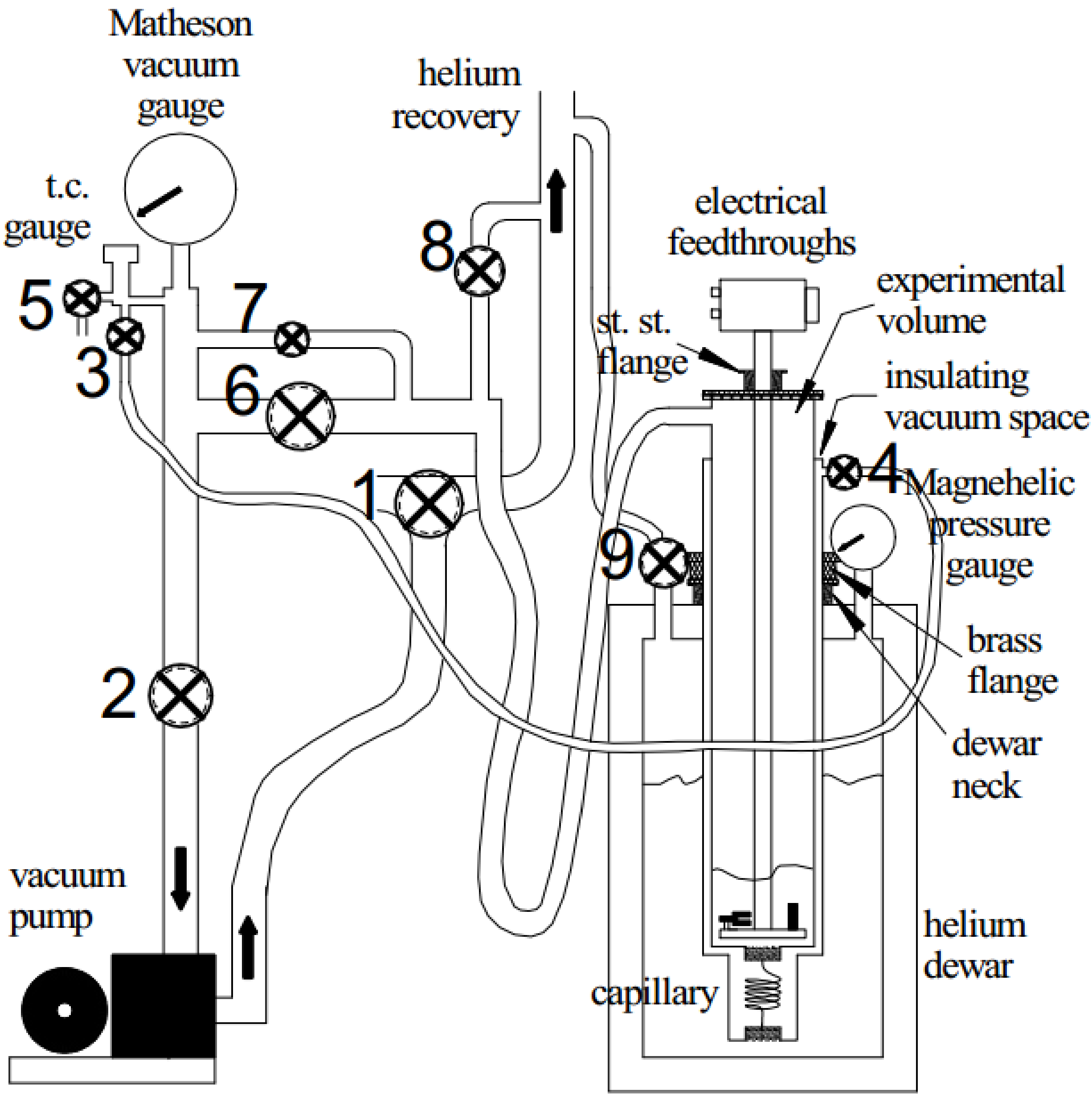

FIG. 12: The gas handling system and how it was a attached to the dewar and ultimately the suck stick.